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Risk Assessment of Raffle-Linked Savings Account Companies: Comparison Between Two Models

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Summary

This paper presents two models for assessment of the raffle risk and calculation of the risk-based capital of companies offering raffle-linked savings plans. We apply these models to a real data set of such a company and compare the results. The models are based on slightly different premises for one of the variables involved, the proportion of tickets not sold at the moment just before each drawing: the first assumes that this proportion, θ, is known, and the second assumes that this proportion, Θ, is a random variable with probability density function $Beta$. The results indicate that the uncertainty over the proportion of unsold tickets does not interfere in the capital requirement to cover the raffle risk and the two models generate identical results when $E[\Theta]$ is equal to θ. Future research avenues include analyses of cost and benefit with relaxation or adoption of other premises.

Key Words

Raffle risk; lottery risk; raffle-linked savings company; capital requirement; risk-based capital.

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Sinopse

Avaliação do Risco de Sorteio das Sociedades de Capitalização: Comparação entre Dois Modelos

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Resumo

Neste artigo, nós apresentamos dois modelos para avaliação do risco de sorteio e cálculo do capital de risco baseado no risco de subscrição das sociedades de capitalização, aplicamos esses modelos a um conjunto real de dados de uma sociedade de capitalização e comparamos os seus resultados. Os modelos adotam premissas ligeiramente diferentes para uma das variáveis envolvidas, a proporção de títulos não vendidos no momento imediatamente anterior à realização de cada sorteio: o primeiro assume que esta proporção, θ, é conhecida; o segundo assume que esta proporção, Θ, é uma variável aleatória com função densidade de probabilidade $Beta$. O artigo mostra que a incerteza sobre a proporção de títulos não vendidos não interfere no requerimento de capital para cobrir o risco de sorteio e os dois modelos geram resultados idênticos quando $E[\Theta]$ é igual a θ. Perspectivas de pesquisa futura incluem análises de custo e benefício com o relaxamento ou a adoção de outras premissas.

Palavras-Chave

Risco de sorteio; risco de loteria; sociedade de capitalização; requerimento de capital; capital baseado em risco.

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Sinopsis

Evaluación del Riesgo de Sorteo de las Sociedades de Capitalización: Comparación entre Dos Modelos

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Resumen

En este artículo, presentamos dos modelos para evaluación de riesgo de sorteo y cálculo del capital de riesgo basado en el riesgo de subscripción de las sociedades de capitalización, aplicamos esos modelos a un conjunto real de datos de una sociedad de capitalización y comparamos sus resultados. Los modelos adoptan premisas ligeramente diferentes para una de las variables involucradas, la proporción de títulos no vendidos en el momento inmediatamente anterior a la realización de cada sorteo: el primero asume que esta proporción, θ, es conocida; el segundo asume que esta proporción, Θ, es una variable aleatoria con función densidad de probabilidad $Beta$. El artículo muestra que la incertidumbre sobre la proporción de títulos no vendidos no interfiere en el requerimiento de capital para cubrir el riesgo de sorteo y los dos modelos generan resultados idénticos cuando $E[\Theta]$ es igual a θ. Perspectivas de pesquisa futura incluyen análisis de costo y beneficio con el relajamiento o la adopción de otras premisas.

Palabras-Clave

Riesgo de sorteo; riesgo de lotería; sociedad de capitalización; requerimiento de capital; capital basado en riesgo.

Sumario

1. Introducción. 2. Métodos de cálculo. 3. Comparación entre los dos modelos. 4. Aplicación en un caso real. 5. Conclusión y perspectivas de pesquisa. 6. Referencias bibliográficas.

1. Introduction

Raffle-linked savings plans have been present in Brazil since 1929. Under these plans, savers are eligible for prizes in products or cash, distributed according to periodic drawings, in return for a lower yield on their savings. There is also generally a penalty for early redemption of the amount deposited before the end of the contract. The deposit is made by the purchase of savings tickets.[1] The raffle-linked savings plans are organized in series, where each series defines the tickets that compete in the same drawings. There is a good deal of flexibility in the regulations, and some of the plans more closely resemble pure lotteries. Raffle-linked savings plans can be of four different modalities (traditional, incentive, popular, programmed-purchase) and three different types (single payment; monthly payment; periodic payment). Raffle-linked savings plans of the same modality and type are subject to the same or similar regulatory requirements.

In this paper we present the approaches proposed in Franklin *et al.* (2012) and Melo *et al.* (2012) for assessment of the raffle risk of raffle-linked savings account companies. We compare these two models and apply them to a set of actual data from a raffle-linked savings company. The value at risk (*VaR*) measure is used to calculate the amount of capital required to ensure that these companies do not become insolvent. In a context where financial institutions are subject to increasingly strict rules on risk management to protect the interests of customers, the adequate measurement of the raffle risk, in simple and direct form, is important to help devise rules to assure the solidity of these companies. This is a focus of the supervision and risk management guidelines of, for example, the International Association of Insurance Supervisors (IAIS), as well as of the European Union through the Solvency II Directive.

2. Calculation methods

Franklin *et al.* (2012) developed a raffle risk measurement model based on a set of premises organized into two groups: one for the proportion of savings tickets not sold for a particular drawing and the other for the raffle risk premium. These premises establish that: (i) the proportion of the tickets not sold at the moment immediately before the drawing, for each future drawing i, of a determined series of a particular savings plan of a determined modality/type is a known value, θ; and (ii) the amount to be paid out in each future drawing i to winning participants (*i.e.*, the value of the prizes), is a random variable, A_i, whose probability density function is unique for each modality/type of plan. The approach developed by Melo *et al.* (2012) partly follows the steps of Franklin *et al.* (2012), but assumes that the proportion of unsold tickets for drawing i, before the raffle date, is a random variable Θ, with *Beta* distribution, whose probability density function is unique for each modality/type of plan.

In both models, the variable I_i is the indicator for the event of a subscriber ticket being chosen in drawing i. In other words, it is a random variable that assumes the value *one* when the number drawn as the winner corresponds to a subscriber ticket, and the value *zero* when the number drawn does not correspond to any subscriber ticket so that the amount reserved for the prize reverts to the company. Table 1 shows the

[1] *Títulos de capitalização* (in Portuguese).

mathematical representation of the variable I_i in each model, together with the formulas for its expected value and variance.

Table 1 – The indicator for the event of a subscriber ticket being chosen in drawing i

Model proposed by Franklin *et al.* (2012):	**Model proposed by Melo *et al.* (2012):**
(1) $I_i = \begin{cases} 1, \text{ with probability } 1\text{-}\theta \\ 0, \text{ with probability } \theta \end{cases}$ Where θ is a known value (deterministic).	(2) $I_i \mid \Theta = \begin{cases} 1, \text{ with probability } 1 - \Theta \\ 0, \text{ with probability } \Theta \end{cases}$ Where Θ is a random variable with probability density function *Beta (α, β)*.
$E[I_i] = (1-\theta)$ $Var[I_i] = \theta(1-\theta)$	$E[I_i] = E[E[I_i \mid \Theta]] = \dfrac{\beta}{\alpha+\beta}$ $Var[I_i] = E[Var[I_i \mid \Theta]] + Var[E[I_i \mid \Theta]]$ $= \dfrac{\alpha\beta}{(\alpha+\beta)^2}$

The value of the raffle prize, A_i, is equal to the amount paid by the winning subscriber for the ticket (vp_i) multiplied by a multiple (specific of each plan): $A_i = vp_i .(multiple)$. The *raffle multiple* is a known quantity so that the source of the randomness of the variable A_i (expense of the company for each future drawing i) comes from the amount paid by the winning subscriber (vp_i), where this payment can fluctuate within the interval $[vp\_min, vp\_max]$.

Let $A_i \mid I_i = 1$ be the raffle prize received by the winning subscriber of drawing i. Two points should be mentioned here. First, the probability density function of A_i is only of interest when one of the contestants is selected in the drawing ($I_i = 1$), because this is when the company will have to pay the amount of the corresponding prize. Second, at the moment the raffle risk is measured, this value is unknown. Therefore, A_i is a random variable that assumes values in the interval $[vp\_min.(multiple), vp\_max.(multiple)]$, where the lower and upper bounds of payment to the winner are characteristics of each modality/type of plan. Its mean and variance are given by:

$$\begin{cases} E[A_i \mid I_i = 1] = \mu_A \\ \\ Var[A_i \mid I_i = 1] = \sigma_A^2 \end{cases} \tag{3}$$

As discussed by Kaas *et al*. (2004), the conditional distribution of $A_i \mid I_i = 0$ is irrelevant, so for convenience we consider it to be equivalent to the distribution of $A_i \mid I_i = 1$. Since the two distributions are equivalent, it can be assumed that A_i and I_i are independent random variables. That fact justifies our consideration that the distribution of A_i is equal to that of $A_i \mid I_i = 1$. Therefore, in view of the relation between A_i and vp_i, we have the following:

$$\begin{cases} E[A_i] = E[A_i \mid I_i = 1] = \mu_A = \mu_{vp}.(multiple) \\ \\ Var[A_i] = Var[A_i \mid I_i = 1] = \sigma_A^2 = \sigma_{vp}^2.(multiple)^2 \end{cases} \tag{4}$$

Now let $DrawPay_i$ be a random variable that represents the payout expense incurred by the company for drawing i (of a determined series of tickets of a particular plan of a determined modality/type). According to the Individual Risk Theory (see, for example, Ferreira, 2002, or Bowers *et al*., 1977), the random variable $DrawPay_i$ can be modeled as the product of two other variables:

$$DrawPay_i = I_i A_i \tag{5}$$

$DrawPay_i$ and $DrawPay_j$ $(i \neq j)$ are independent random variables: the result of one drawing does not change the probabilities of other drawings. Considering that *NFD* is the number of future drawings to be realized for all the series and plans of a determined modality/type of plan, according to the Central Limit Theorem (CLT), for a sufficiently large number of drawings,[2] the probability density function of $\sum_{i=1}^{NFD} DrawPay_i$ can be approximated by a *Normal* distribution.

The net loss of a raffle-linked savings company from all the future drawings of all plans and series of a determined modality/type of plan, $NetLoss$, is equal to the difference between the amount paid out by the company to the winners of all these drawings, $\sum_{i=1}^{NFD} DrawPay_i$, and the total amount taken in by the company as provision for future drawings, *ProvFD*.

$$NetLoss = \sum_{i=1}^{NFD} DrawPay_i - ProvFD \tag{6}$$

The amount of the provision for future drawings, from the way the provision is constituted, is always equal to the expected value to be paid by the company to the winners of these drawing (for details, see Franklin *et al*., 2012). Therefore:

$$E[NetLoss] = E[\sum_{i=1}^{NFD} DrawPay_i] - ProvFD = 0 \tag{7}$$

[2] Usually, for $NFD \geq 30$.

$$Var[NetLoss] = Var[\sum_{i=1}^{NFD} DrawPay_i] = \sum_{i=1}^{NFD} Var[DrawPay_i] \quad (8)$$

Table 2 shows the formulas derived by each model to calculate the expected value and variance of the net loss of a raffle-linked savings company from all the drawings of all plans and series of a determined modality/type of plan.

Table 2 – Expected value and variance of the random variable *NetLoss*

Model proposed by Franklin *et al.* (2012):	**Model proposed by Melo *et al.* (2012):**
(9) $$\begin{cases} E[DrawPay_i] = E[E[I_i A_i \mid I_i]] = \mu_A(1-\theta) \\ Var[DrawPay_i] = Var[E[I_i A_i \mid I_i]] + \\ + E[Var[I_i A_i \mid I_i]] = \mu_A^2(1-\theta)\theta + \sigma_A^2(1-\theta) \end{cases}$$	(10) $$\begin{cases} E[DrawPay_i] = \mu_A\left(1 - \frac{\alpha}{\alpha+\beta}\right) \\ Var[DrawPay_i] = \mu_A^2 \frac{\alpha\beta}{(\alpha+\beta)^2} + \sigma_A^2 \frac{\beta}{\alpha+\beta} \end{cases}$$
(11) $$\begin{cases} E[NetLoss] = 0 \\ Var[NetLoss] = NFD.[\mu_A^2(1-\theta)\theta + \sigma_A^2(1-\theta)] \end{cases}$$	(12) $$\begin{cases} E[NetLoss] = 0 \\ Var[NetLoss] = NFD\left(\mu_A^2 \frac{\alpha\beta}{(\alpha+\beta)^2} + \sigma_A^2 \frac{\beta}{\alpha+\beta}\right) \end{cases}$$

Where:

- θ is the proportion of unsold tickets just before drawing i (a value assumed known in Franklin *et al.*, 2012);
- μ_A is the expected prize payout (= $E[A_i]$);
- σ_A^2 is the variance of the prize (= $Var[A_i]$);
- NFD is the number of future drawings to be realized for all the series and plans of a determined modality/type of plan;
- α and β are the parameters of the distribution $Beta(\alpha, \beta)$, which models the proportion of unsold tickets (i.e., the random variable Θ in Melo *et al.*, 2012) just before drawing i.

From the mean and variance of the random variable *NetLoss*, obtained from the above equations, one can evaluate the risk exposure of the company from all the future drawings to be held. For this purpose we use the value at risk (*VaR*) measure, defined as follows:

$$VaR_\alpha(NetLoss) = \inf\{z \in R : \Pr(NetLoss > z) \leq \alpha\}, 0 < \alpha < 1.$$

The capital necessary to cover the raffle risk associated with the raffle-linked savings plans of a determined modality/type of a particular company is given by:

$VaR_{\alpha}(NetLoss) - E[NetLoss] = Z_{1-\alpha}.StD(NetLoss)$, where $Z_{1-\alpha}$ is the *quantile* $(1-\alpha)$ of the standard *Normal* distribution, and *StD* is the abbreviation for *standard deviation*.

The total capital requirement to cover all the raffle risk of a raffle-linked savings company is the result of aggregating the capital amounts required to cover the raffle risk of the different modalities/types of plans.

3. Comparison between the two models

The proportion of unsold tickets at the moment just before a future drawing, Θ, is in fact an unknown value at the moment of measuring the raffle risk, so that it is reasonable to assume that Θ is a random variable with distribution $Beta\ (\alpha, \beta)$. The domain of that distribution function is exactly in the interval [0, 1] and its parameterization provides great flexibility in modeling the proportion of tickets sold, because depending on the parameters employed, the distribution $Beta\ (\alpha, \beta)$ can model low or high mean values of the proportion with low or high variability around the mean.

If $\Theta \sim Beta\ (\alpha, \beta)$, then:

$$E[\Theta] = \frac{1}{1+\beta/\alpha} \tag{13}$$

If we set $\theta = E[\Theta]$ and substitute equation (13) in equation (11), we get exactly equation (12), and the models proposed by Franklin *et al.* (2012) and Melo *et al.* (2012) become identical.

This result may at first glance appear counterintuitive, because one would normally expect the need for capital to be greater the higher the uncertainty is. However, it can be explained by observing that the variance of the random variable I_i depends only on $E[\Theta]$, and is not affected by the variance of Θ.

By the variance decomposition formula:

$$Var[I_i] = E[Var[I_i \mid \Theta]] + Var[E[I_i \mid \Theta]]$$
$$= E[\Theta.(1-\Theta)] + Var(1-\Theta) = E[\Theta] - E[\Theta^2] + E[\Theta^2] - E[\Theta]^2$$
$$= E[\Theta] - E[\Theta]^2$$

If $\theta = E[\Theta]$, then $E[I_i] = (1-\theta)$, $Var(I_i) = \theta(1-\theta)$ and the first- and second-order moments of I_i become identical to the case when the proportion of unsold tickets is assumed to be known (deterministic) and equal to θ.

4. Application to a real case

Using a set of actual data from a particular raffle-linked savings company (name not revealed for reasons of confidentiality), we estimated the parameters of the probability density function of the proportion of unsold tickets, as well as the mean and variance of the prize payouts. The estimated parameters are shown in Table 3.[3] The results for the capital requirement to cover the raffle risk for two distinct values of the number of future drawings are shown in Table 4. It can be easily seen that any changes in the parameters α and β, while holding $E[\Theta]$ equal to the sample mean of 21.9% (and allowing the variance of Θ to vary as a function of α and β), will not change the capital requirement.

Table 3 – Estimation of the parameters

Parameter	Estimated point value
α	1.57
β	5.59
$E[\Theta]$	21.9%
$E[A_i]$	$ 755,233
$StD[A_i]$	$ 526,836

Table 4 – Capital requirement

Model	Capital requirement to cover the risk of future drawings at the critical level of 99% (in thousands)	
	NFD = 100	NFD = 500
Franklin *et al.* (2012) or Melo *et al.* (2012)	$ 13,063	$ 29,211

5. Conclusion and research perspectives

This paper shows that as long as there is a good estimate for $E[\Theta]$, the probability density function of the random variable Θ (*i.e.*, the uncertainty over the proportion of unsold tickets) is not relevant for the calculation of the capital requirement to cover the risk of future drawings.

Future research perspectives include cost-benefit analysis for adopting or eliminating other premises of the model – *i.e.*, the analysis of tradeoff between model's improvement and information cost.

[3] The parameters were estimated by the method of moments. We used a sample of 90 observations of proportions of tickets not sold at the moment of the drawing, of the *traditional* modality, in 2010, and another sample of 65 prize values paid in this same modality during the same year.

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